# More About AGM Revision in Description Logics

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# Outline of topics

1. Belief Dynamics
   - Belief Sets
   - Operations
   - AGM Revision
   - AGM Postulates
   - Levi identity

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# Part I

## Introduction to AGM Theory

# Belief Set

AGM theory defines operations on *belief sets*:

**Definition (Belief Set)**

A *belief set* is a set of sentences $K \subseteq \mathscr{L}$ such that $K = Cn(K)$.

# Operations on belief sets

There are three main operations that can be applied to belief sets:

- **Expansion:** is used when the agent has to **add** new information to its belief set.
- **Revision:** is used when the agent has to **add** new information to its belief set in a **consistent way**.
- **Contraction:** is used when an agent wants to **remove** sentences from its belief set.

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# Example of a revision

**Example:**

I thought that no Portuguese writer had won the Nobel prize. I couple of months ago I saw in the newspaper: "Nobel-winning author Jose Saramago dies at 87". Since I believed that Saramago was Portuguese, I had to revise my believes.

To avoid inconsistencies I had to drop my believe about nobel-wining Portuguese writers, or about the nationality of Saramago. I chose the first one.

# Formalizing the revision operation

The belief set of agent, whose belief set was $K$, after contracting $\alpha$ will be represented as $K * \alpha$

Which properties this operation ($K * \alpha$) should satisfy?

Which sets of formulas are candidates for $K * \alpha$?

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# AGM postulates

Alchourrón, Gärdenfors and Makinson (AGM) stated that a revision should satisfy the following postulates:

- **success:** $\alpha \in K * \alpha$
- **closure:** $K * \alpha = Cn(K * \alpha)$
- **inclusion:** $K * \alpha \subseteq K + \alpha$
- **vacuity:** If $K + \alpha$ is consistent then $K * \alpha = K + \alpha$
- **extensionality:** If $Cn(\alpha) = Cn(\beta)$ then $K * \alpha = K * \beta$
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# Levi identity

The postulates don't tell how to **construct** the contraction.

Given that we know how to construct a contraction (e.g. using partial meet construction) then we can use the Levi identity to construct the revision.

**Levi identity**

$$K * \alpha = K - \neg\alpha + \alpha$$

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# Representation Theorem

**Theorem**

*If* $-$ *is a partial meet contraction then* $K - \neg\alpha + \alpha$ *satisfies all the AGM postulates for revision. Furthermore, if* $K * \alpha$ *satisfies all the AGM postulates for revision then there is a partial meet contraction* $-$ *such that* $K * \alpha = K - \neg\alpha + \alpha$

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# Motivation

The representation theorem for revision holds for logics that satisfies certain properties.

# A generic framework for logics

We will consider a logic as a tuple $\langle \mathscr{L}, Cn \rangle$ where:

- $\mathscr{L}$ is the *language* of the logic i.e.: the set of well formed formulas.
- $Cn$ is a *consequence operator* that given a set of formulas return its consequences ($Cn : 2^{\mathscr{L}} \rightarrow 2^{\mathscr{L}}$).

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