

First Steps in \mathcal{EL} Contraction

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Outline

1 Preliminaries

- Belief Change
- Description Logic \mathcal{EL}

2 Contraction in \mathcal{EL}

- First Attempt
- A More Fine-grained Approach
- Other Types of Contraction

3 Conclusion

- Summary, Open questions and Further Work

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Revision, Expansion and Contraction

- Expansion: $K + \varphi$
- Revision: $K \star \varphi$, with $K \star \varphi \models \varphi$ and $K \star \varphi \not\models \perp$
- Contraction: $K - \varphi \not\models \varphi$

Revision, Expansion and Contraction

- Expansion: $K + \varphi$
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- Contraction: $K - \varphi \not\models \varphi$

Also meaningful for ontologies



AGM Approach

Contraction described on the *knowledge level*

Rationality Postulates

$$(K-1) \quad K - \varphi = \text{Cn}(K - \varphi)$$

$$(K-2) \quad K - \varphi \subseteq K$$

$$(K-3) \quad \text{If } \varphi \notin K, \text{ then } K - \varphi = K$$

$$(K-4) \quad \text{If } \not\models \varphi, \text{ then } \varphi \notin K - \varphi$$

$$(K-5) \quad \text{If } \varphi \equiv \psi, \text{ then } K - \varphi = K - \psi$$

$$(K-6) \quad \text{If } \varphi \in K, \text{ then } (K - \varphi) + \varphi = K$$

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AGM Approach

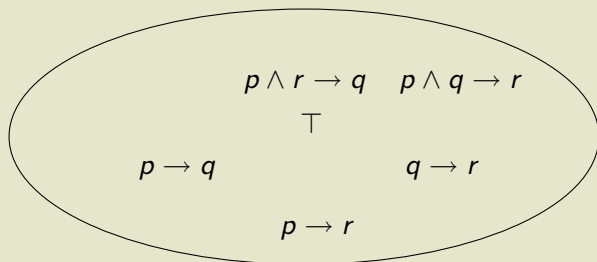
Construction method:

- Identify the maximally consistent subsets that do not entail φ (remainder sets)
- Pick *some* non-empty subset of remainder sets
 - ▶ Take their intersection: Partial meet
- Pick *all* remainder sets: Full meet
- Pick a *single* remainder set: Maxichoice

AGM Approach

Example

Contraction of $\{p \rightarrow r\}$ from Horn theory $K = Cn(\{p \rightarrow q, q \rightarrow r\})$

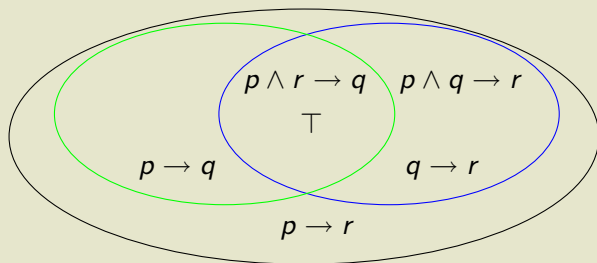


- Maxichoice?
- Full meet?

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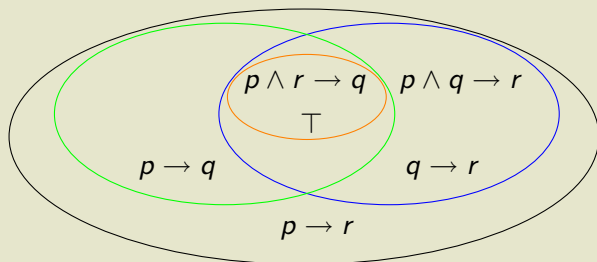


- Maxichoice? $H_{mc}^1 = Cn(\{p \rightarrow q\})$ or $H_{mc}^2 = Cn(\{q \rightarrow r, p \wedge r \rightarrow q\})$
- Full meet?

AGM Approach

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- Maxichoice? $H_{mc}^1 = Cn(\{p \rightarrow q\})$ or $H_{mc}^2 = Cn(\{q \rightarrow r, p \wedge r \rightarrow q\})$
- Full meet? $H_{fm} = Cn(\{p \wedge r \rightarrow q\})$

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Description Logic \mathcal{EL} [Baader, 2003] with \perp

- Concepts

$$C ::= A \mid \top \mid \perp \mid C \sqcap C \mid \exists R.C$$

- Interpretations $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$

$$A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}, \quad R^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}},$$

$$\top^{\mathcal{I}} = \Delta^{\mathcal{I}}, \quad \perp^{\mathcal{I}} = \emptyset,$$

$$(C \sqcap D)^{\mathcal{I}} = C^{\mathcal{I}} \cap D^{\mathcal{I}},$$

$$(\exists R.C)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} \mid \exists b.(a, b) \in R^{\mathcal{I}} \text{ and } b \in C^{\mathcal{I}}\}$$

Description Logic \mathcal{EL} [Baader, 2003]

- Axioms $C \sqsubseteq D$

$$\mathcal{I} \models C \sqsubseteq D \text{ iff } C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$$

- TBox \mathcal{T} : set of axioms

$$\mathcal{I} \models \mathcal{T} \text{ iff } \mathcal{I} \text{ satisfies every axiom in } \mathcal{T}$$

- $\mathcal{T} \models C \sqsubseteq D$ iff for all \mathcal{I} if $\mathcal{I} \models \mathcal{T}$ then $\mathcal{I} \models C \sqsubseteq D$
- $Cn(\mathcal{T}) = \{C \sqsubseteq D \mid \mathcal{T} \models C \sqsubseteq D\}$

Description Logic \mathcal{EL} [Baader, 2003]

Example

$\mathcal{T} = \text{Cn}(\{A \sqsubseteq B, B \sqsubseteq \exists R.A\})$

$A \sqsubseteq B$

$A \sqcap \exists R.A \sqsubseteq B$

\top

$A \sqcap B \sqsubseteq \exists R.A$

$B \sqsubseteq \exists R.A$

$A \sqsubseteq \exists R.A$

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Motivation

Let \mathcal{T} be a TBox and Φ be a set of axioms

- Contract \mathcal{T} with Φ
 - ▶ we want $\mathcal{T} \not\models \Phi$
 - ▶ Some axiom in Φ should not follow from \mathcal{T} anymore

Following Delgrande's Approach [KR'2008]

Definition (Remainder Sets)

For a belief set \mathcal{T} , $X \in \mathcal{T} \downarrow \Phi$ iff

- $X \subseteq \mathcal{T}$
- $X \not\models \Phi$
- for every X' s.t. $X \subset X' \subseteq \mathcal{T}$, $X' \models \Phi$.

• We call $\mathcal{T} \downarrow \Phi$ the *remainder sets* of \mathcal{T} w.r.t. Φ

• Do they exist?

▶ \mathcal{EL} is compact and has a Tarskian consequence relation

Definition (Selection Functions)

A selection function σ is a function from $\mathcal{P}(\mathcal{P}(\mathcal{L}_{\mathcal{EL}}))$ to $\mathcal{P}(\mathcal{P}(\mathcal{L}_{\mathcal{EL}}))$ s.t. $\sigma(\mathcal{T} \downarrow \Phi) = \{\mathcal{T}\}$ if $\mathcal{T} \downarrow \Phi = \emptyset$, and $\sigma(\mathcal{T} \downarrow \Phi) \subseteq \mathcal{T} \downarrow \Phi$ otherwise.

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Following Delgrande's Approach

Definition (Partial Meet Contraction)

Given a selection function σ , $-_{\sigma}$ is a partial meet contraction iff

$$\mathcal{T}_{-_{\sigma}} \Phi = \bigcap \sigma(\mathcal{T} \downarrow \Phi).$$

Definition (Maxichoice and Full Meet)

Given a selection function σ , $-_{\sigma}$ is a *maxichoice contraction* iff $\sigma(\mathcal{T} \downarrow \Phi)$ is a singleton set. It is a *full meet contraction* iff $\sigma(\mathcal{T} \downarrow \Phi) = \mathcal{T} \downarrow \Phi$ when $\mathcal{T} \downarrow \Phi \neq \emptyset$.

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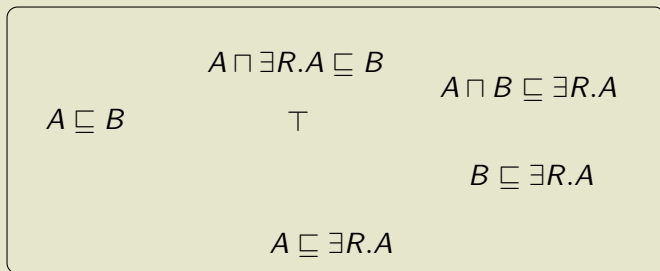
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Following Delgrande's Approach

Example

Contraction of $\{A \sqsubseteq \exists R.A\}$ from $\mathcal{T} = \text{Cn}(\{A \sqsubseteq B, B \sqsubseteq \exists R.A\})$

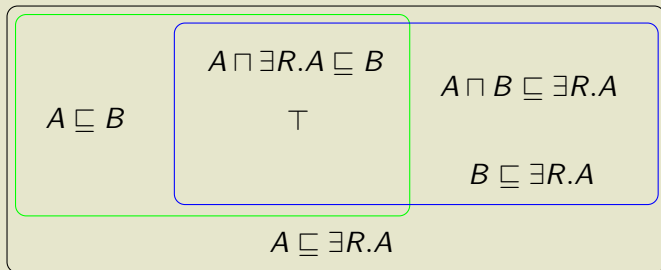


- Maxichoice?
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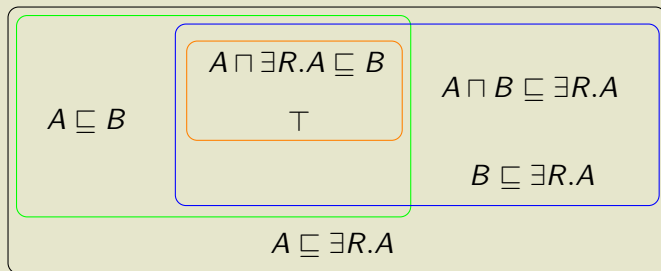


- Maxichoice? $\mathcal{T}_{mc}^1 = Cn(\{A \sqsubseteq B\})$ or $\mathcal{T}_{mc}^2 = Cn(\{B \sqsubseteq \exists R.A, A \sqsubseteq \exists R.A \sqsubseteq B\})$
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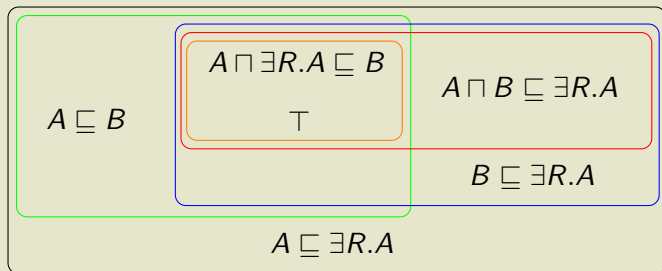


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- Full meet? $\mathcal{T}_{fm} = Cn(\{A \sqsubseteq \exists R.A \sqsubseteq B\})$

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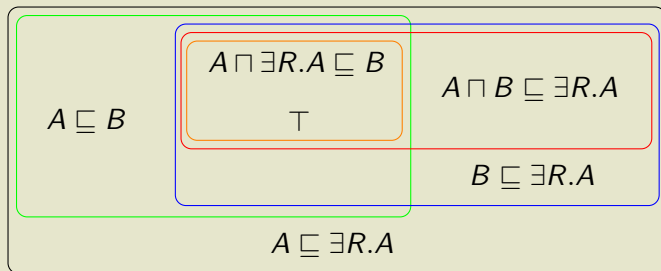


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- What about $\mathcal{T}' = Cn(\{A \sqsubseteq \exists R.A \sqsubseteq B, A \sqsubseteq B \sqsubseteq \exists R.A\})$?

Following Delgrande's Approach

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- Full meet? $\mathcal{T}_{fm} = \text{Cn}(\{A \sqsubseteq \exists R.A \sqsubseteq B\})$
- What about $\mathcal{T}' = \text{Cn}(\{A \sqsubseteq \exists R.A \sqsubseteq B, A \sqsubseteq B \sqsubseteq \exists R.A\})$?
- $\mathcal{T}_{fm} \subseteq \mathcal{T}' \subseteq \mathcal{T}_{mc}^2$, but there is no partial meet contraction yielding \mathcal{T}' !

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Beyond Partial Meet [Booth *et al.*, IJCAI'09]

Definition (Infra-Remainder Sets)

For belief sets \mathcal{T} and X , $X \in \mathcal{T} \downarrow \Phi$ iff there is some $X' \in \mathcal{T} \downarrow \Phi$ s.t.
 $(\bigcap \mathcal{T} \downarrow \Phi) \subseteq X \subseteq X'$.

We call $\mathcal{T} \downarrow \Phi$ the *infra-remainder sets* of \mathcal{T} w.r.t. Φ .

Infra-remainder sets contain *all* belief sets between some remainder set and the intersection of all remainder sets

Beyond Partial Meet [Booth *et al.*, IJCAI'09]

Definition (\mathcal{EL} Contraction)

An infra-selection function τ is a function from $\mathcal{P}(\mathcal{P}(\mathcal{L}_{\mathcal{EL}}))$ to $\mathcal{P}(\mathcal{L}_{\mathcal{EL}})$ s.t. $\tau(\mathcal{T} \Downarrow \Phi) = \mathcal{T}$ whenever $\models \Phi$, and $\tau(\mathcal{T} \Downarrow \Phi) \in \mathcal{T} \Downarrow \Phi$ otherwise. A contraction function $-_{\tau}$ is an \mathcal{EL} -contraction iff $\mathcal{T} -_{\tau} \Phi = \tau(\mathcal{T} \Downarrow \Phi)$.

A Representation Result

Basic postulates for \mathcal{EL} contraction

$$(T-1) \quad T - \Phi = Cn(T - \Phi)$$

$$(T-2) \quad T - \Phi \subseteq T$$

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$$(T-6) \quad \text{If } \varphi \in T \setminus (T - \Phi) \text{ then there is a } T' \text{ such that} \\ \bigcap(T \downarrow \Phi) \subseteq T' \subseteq T, T' \not\models \Phi, \text{ and } T' + \{\varphi\} \models \Phi$$

Conjecture

Every \mathcal{EL} contraction satisfies (T-1)–(T-6). Conversely, every contraction function satisfying (T-1)–(T-6) is an \mathcal{EL} contraction.

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Other Types of Contraction

Inconsistency-based Contraction [Delgrande, KR'2008]

Let \mathcal{T} be a TBox and Φ be a set of axioms

- Contract \mathcal{T} 'making room' for Φ
- We want $\mathcal{T}' + \Phi \not\equiv \perp$

Package Contraction [Booth *et al.*, IJCAI'09]

Let \mathcal{T} be a TBox and Φ be a set of axioms

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Summary

- Basic AGM account of contraction for \mathcal{EL}
- Weaker than partial meet contraction

Open questions

- Are infra-remainder sets enough?
- Is $C_n(\cdot)$ what we really want?
- Kernel contraction? (Renata knows the answer 😊)
- What about the syntax? (A , $A \sqcap \exists R.A$, $A \sqcap \exists R.\exists R.A$, ...)

Current and Future Work

- Answer questions above
- Full AGM setting: extended postulates
- Relation to justifications in ontology repair

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