First Steps in $\mathcal{EL}$ Contraction

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Outline

1. Preliminaries
   - Belief Change
   - Description Logic $\mathcal{EL}$
2. Contraction in $\mathcal{EL}$
   - First Attempt
   - A More Fine-grained Approach
   - Other Types of Contraction
3. Conclusion
   - Summary, Open questions and Further Work

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Revision, Expansion and Contraction

- Expansion: $K + \varphi$
- Revision: $K \star \varphi$, with $K \star \varphi \models \varphi$ and $K \star \varphi \not\models \bot$
- Contraction: $K - \varphi \not\models \varphi$

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- Contraction: $K - \varphi \not\models \varphi$

Also meaningful for ontologies

AGM Approach

Contraction described on the *knowledge level*

Rationality Postulates

(K–1) $K - \varphi = Cn(K - \varphi)$

(K–2) $K - \varphi \subseteq K$

(K–3) If $\varphi \notin K$, then $K - \varphi = K$

(K–4) If $\not\models \varphi$, then $\varphi \notin K - \varphi$

(K–5) If $\varphi \equiv \psi$, then $K - \varphi = K - \psi$

(K–6) If $\varphi \in K$, then $(K - \varphi) + \varphi = K$

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AGM Approach

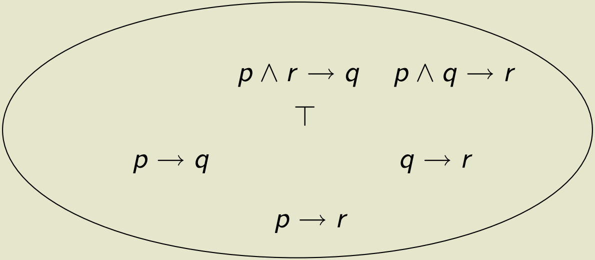
Construction method:

- Identify the maximally consistent subsets that do not entail φ (remainder sets)
- Pick *some* non-empty subset of remainder sets
  - Take their intersection: Partial meet
- Pick *all* remainder sets: Full meet
- Pick a *single* remainder set: Maxichoice

AGM Approach

Example

Contraction of $\{p \rightarrow r\}$ from Horn theory $K = Cn(\{p \rightarrow q, q \rightarrow r\})$



- Maxichoice?
- Full meet?

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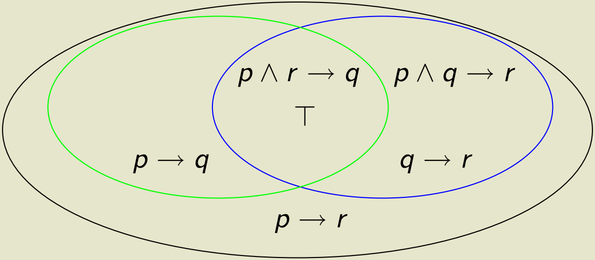

- Maxichoice? $H^1_{mc} = Cn(\{p \rightarrow q\})$ or $H^2_{mc} = Cn(\{q \rightarrow r, p \wedge r \rightarrow q\})$
- Full meet?

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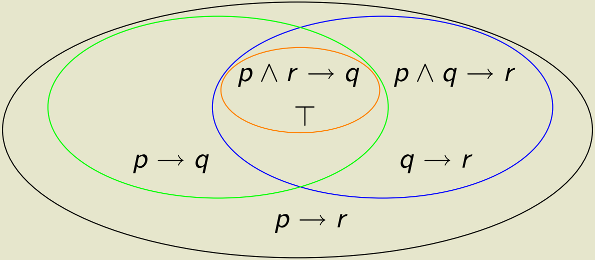

- Maxichoice? $H^1_{mc} = Cn(\{p \rightarrow q\})$ or $H^2_{mc} = Cn(\{q \rightarrow r, p \wedge r \rightarrow q\})$
- Full meet? $H_{fm} = Cn(\{p \wedge r \rightarrow q\})$

Outline

Description Logic $\mathcal{EL}$ [Baader, 2003] with $\bot$

- Concepts

$$C ::= A \mid \top \mid \bot \mid C \sqcap C \mid \exists R.C$$

- Interpretations $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$

$$A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}},\ R^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}},$$

$$\top^{\mathcal{I}} = \Delta^{\mathcal{I}},\ \bot^{\mathcal{I}} = \emptyset,$$

$$(C \sqcap D)^{\mathcal{I}} = C^{\mathcal{I}} \cap D^{\mathcal{I}},$$

$$(\exists R.C)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} \mid \exists b.(a, b) \in R^{\mathcal{I}} \text{ and } b \in C^{\mathcal{I}}\}$$

Description Logic $\mathcal{EL}$ [Baader, 2003]

- Axioms $C \sqsubseteq D$

$$\mathcal{I} \models C \sqsubseteq D \text{ iff } C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$$

- TBox $\mathcal{T}$: set of axioms

$$\mathcal{I} \models \mathcal{T} \text{ iff } \mathcal{I} \text{ satisfies every axiom in } \mathcal{T}$$

- $\mathcal{T} \models C \sqsubseteq D$ iff for all $\mathcal{I}$ if $\mathcal{I} \models \mathcal{T}$ then $\mathcal{I} \models C \sqsubseteq D$
- $Cn(\mathcal{T}) = \{C \sqsubseteq D \mid \mathcal{T} \models C \sqsubseteq D\}$

Description Logic $\mathcal{EL}$ [Baader, 2003]

Example

$\mathcal{T} = Cn(\{A \sqsubseteq B, B \sqsubseteq \exists R.A\})$

$A \sqcap \exists R.A \sqsubseteq B$

$A \sqcap B \sqsubseteq \exists R.A$

$A \sqsubseteq B$

$\top$

$B \sqsubseteq \exists R.A$

$A \sqsubseteq \exists R.A$

Outline

Motivation

Let $\mathcal{T}$ be a TBox and Φ be a set of axioms

- Contract $\mathcal{T}$ with Φ
  - we want $\mathcal{T} \not\models \Phi$
  - Some axiom in Φ should not follow from $\mathcal{T}$ anymore

Following Delgrande's Approach [KR'2008]

Definition (Remainder Sets)

For a belief set $\mathcal{T}$, $X \in \mathcal{T}{\downarrow}\Phi$ iff

- $X \subseteq \mathcal{T}$
- $X \not\models \Phi$
- for every X' s.t. $X \subset X' \subseteq \mathcal{T}$, $X' \models \Phi$.

- We call $\mathcal{T}{\downarrow}\Phi$ the *remainder sets* of $\mathcal{T}$ w.r.t. Φ
- Do they exist?
  - $\mathcal{EL}$ is compact and has a Tarskian consequence relation

Definition (Selection Functions)

A selection function σ is a function from $\mathscr{P}(\mathscr{P}(\mathcal{L}_{\mathcal{EL}}))$ to $\mathscr{P}(\mathscr{P}(\mathcal{L}_{\mathcal{EL}}))$ s.t. $\sigma(\mathcal{T}{\downarrow}\Phi) = \{\mathcal{T}\}$ if $\mathcal{T}{\downarrow}\Phi = \emptyset$, and $\sigma(\mathcal{T}{\downarrow}\Phi) \subseteq \mathcal{T}{\downarrow}\Phi$ otherwise.

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Definition (Partial Meet Contraction)

Given a selection function σ, $-_\sigma$ is a partial meet contraction iff $\mathcal{T} -_\sigma \Phi = \bigcap \sigma(\mathcal{T}{\downarrow}\Phi)$.

Definition (Maxichoice and Full Meet)

Given a selection function σ, $-_\sigma$ is a *maxichoice contraction* iff $\sigma(\mathcal{T}{\downarrow}\Phi)$ is a singleton set. It is a *full meet contraction* iff $\sigma(\mathcal{T}{\downarrow}\Phi) = \mathcal{T}{\downarrow}\Phi$ when $\mathcal{T}{\downarrow}\Phi \neq \emptyset$.

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Following Delgrande's Approach

Example

Contraction of $\{A \sqsubseteq \exists R.A\}$ from $\mathcal{T} = Cn(\{A \sqsubseteq B, B \sqsubseteq \exists R.A\})$

$A \sqcap \exists R.A \sqsubseteq B$

$A \sqcap B \sqsubseteq \exists R.A$

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$B \sqsubseteq \exists R.A$

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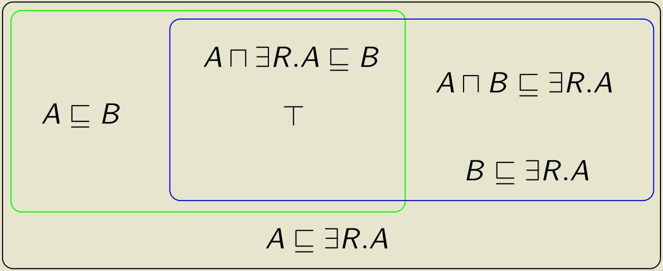

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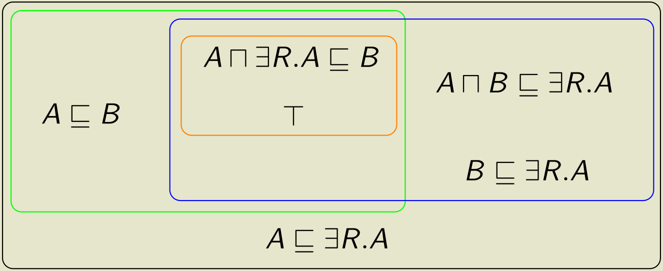

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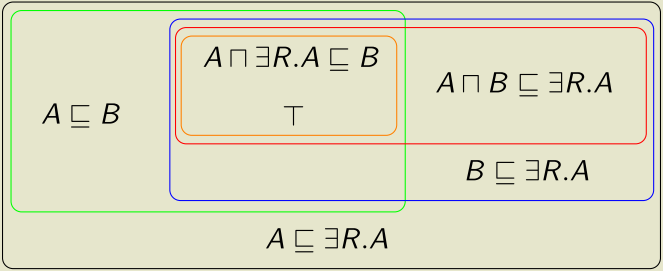

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- What about $\mathcal{T}' = Cn(\{A \sqcap \exists R.A \sqsubseteq B, A \sqcap B \sqsubseteq \exists R.A\})$?

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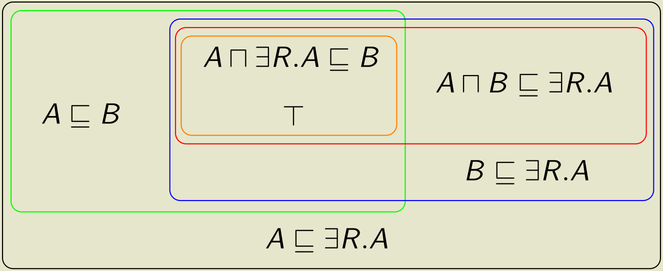

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- $\mathcal{T}_{fm} \subseteq \mathcal{T}' \subseteq \mathcal{T}^2_{mc}$, but there is no partial meet contraction yielding $\mathcal{T}'$!

Outline

Beyond Partial Meet [Booth *et al.*, IJCAI'09]

Definition (Infra-Remainder Sets)

For belief sets $\mathcal{T}$ and X, $X \in \mathcal{T}{\Downarrow}\Phi$ iff there is some $X' \in \mathcal{T}{\downarrow}\Phi$ s.t. $(\bigcap \mathcal{T}{\downarrow}\Phi) \subseteq X \subseteq X'$.

We call $\mathcal{T}{\Downarrow}\Phi$ the *infra-remainder sets* of $\mathcal{T}$ w.r.t. Φ.

Infra-remainder sets contain *all* belief sets between some remainder set and the intersection of all remainder sets

Beyond Partial Meet [Booth *et al.*, IJCAI'09]

Definition ($\mathcal{EL}$ Contraction)

An infra-selection function τ is a function from $\mathscr{P}(\mathscr{P}(\mathcal{L}_{\mathcal{EL}}))$ to $\mathscr{P}(\mathcal{L}_{\mathcal{EL}})$ s.t. $\tau(\mathcal{T}\Downarrow\Phi) = \mathcal{T}$ whenever $\models \Phi$, and $\tau(\mathcal{T}\Downarrow\Phi) \in \mathcal{T}\Downarrow\Phi$ otherwise. A contraction function $-_\tau$ is an $\mathcal{EL}$-contraction iff $\mathcal{T} -_\tau \Phi = \tau(\mathcal{T}\Downarrow\Phi)$.

A Representation Result

Basic postulates for $\mathcal{EL}$ contraction

($\mathcal{T}-1$) $\mathcal{T}-\Phi = Cn(\mathcal{T}-\Phi)$

($\mathcal{T}-2$) $\mathcal{T}-\Phi \subseteq \mathcal{T}$

($\mathcal{T}-3$) If $\Phi \not\subseteq \mathcal{T}$ then $\mathcal{T}-\Phi = \mathcal{T}$

($\mathcal{T}-4$) If $\not\models \Phi$ then $\Phi \not\subseteq \mathcal{T}-\Phi$

($\mathcal{T}-5$) If $Cn(\Phi) = Cn(\Psi)$ then $\mathcal{T}-\Phi = \mathcal{T}-\Psi$

($\mathcal{T}-6$) If $\varphi \in \mathcal{T} \setminus (\mathcal{T}-\Phi)$ then there is a $\mathcal{T}'$ such that $\bigcap(\mathcal{T}{\downarrow}\Phi) \subseteq \mathcal{T}' \subseteq \mathcal{T}$, $\mathcal{T}' \not\models \Phi$, and $\mathcal{T}' + \{\varphi\} \models \Phi$

Conjecture

Every $\mathcal{EL}$ contraction satisfies ($\mathcal{T}-1$)–($\mathcal{T}-6$). Conversely, every contraction function satisfying ($\mathcal{T}-1$)–($\mathcal{T}-6$) is an $\mathcal{EL}$ contraction.

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Other Types of Contraction

Inconsistency-based Contraction [Delgrande, KR'2008]

Let $\mathcal{T}$ be a TBox and Φ be a set of axioms

- Contract $\mathcal{T}$ 'making room' for Φ
- We want $\mathcal{T}' + \Phi \not\models \bot$

Package Contraction [Booth *et al.*, IJCAI'09]

Let $\mathcal{T}$ be a TBox and Φ be a set of axioms

- Contract $\mathcal{T}$ so that *none* of the axioms in Φ follows from it
- Removal of *all* sentences in Φ from $\mathcal{T}$

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Summary, Open questions and Further Work

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- Basic AGM account of contraction for $\mathcal{EL}$
- Weaker than partial meet contraction

Open questions

- Are infra-remainder sets enough?
- Is $Cn(.)$ what we really want?
- Kernel contraction? (Renata knows the answer ☺)
- What about the syntax? (A, $A \sqcap \exists R.A$, $A \sqcap \exists R.\exists R.A$, ...)

Current and Future Work

- Answer questions above
- Full AGM setting: extended postulates
- Relation to justifications in ontology repair

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